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ON THE STABILIZATION OF CERTAIN NON-LINEAR SYSTEMS*

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The problem of the stabilization of systems with a non-linearity, dependent on a small parameter ε, is studied. A quasi-optimal stabilization algorithm is proposed and substantiated for the case of small ε. If nothing is known regarding the magnitude of ε, the technique of adaptive stabilization is developed. Examples of synthesis in the control of the motion of robots with unknown parameters are considered.

The problem of the stabilization of motions, with which a large number of investigations have been concerned, is studied in two formulations /1/. The first is associated with the determination of the control under which the system becomes stable while the second is associated with the choice of the control which minimizes a functional (the quality criterion). Generally speaking, the above-mentioned formulations are not equivalent. In order that the control which minimizes the integral quality criterion should simultaneously make the system stable, it follows that one should consider quality criteria which are positive-definite with respect to the phase coordinates (quadratic criteria, for example). Moreover, if the perturbations in the system are small, then a non-linear system may be approximated by a linear system. In fact, in the case of problems involving the stabilization of linear systems, the final results have been obtained with a quadratic quality criterion.

The question of whether it is possible to expand the Bellman function and the optimal control in power series in small perturbations and the convergence of these series has been investigated in /3, 4/ for non-linear problems and small perturbations. At the same time, the initial perturbations may not be small when real systems are treated and it is therefore necessary to take account of non-linearity when constructing the control.

A method of quasi-optimal stabilization is presented below and error estimates are obtained for the case of arbitrary initial perturbations.

1. Formulation of the quasi-optimal stabilization problem. A control system has the form

$$x^{\cdot}(t) = \varepsilon f(t, x(t)) + B(t)u, \quad t \geqslant 0, \quad x(0) = x_0 \tag{1.1}$$

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Here, the vector $x \in R_n$ (where R_n is an n-dimensional Euclidean space), the function $f(t, x)$, which is measurable with respect to its set of arguments, satisfies a local Lipshitz condition with respect to x, $|f(t, x)| \leqslant Cx$, the matrix $B(t)$ is measurable and bounded in the interval $[0, \infty)$ and the parameter, $\varepsilon \geqslant 0$ and the vector x_0 are specified. Here and subsequently, various positive constants are denoted by the letter C. The control $u = u(t, x(t))$ has to be chosen in such a way that the functional

$$J(u) = \int_0^\infty (x'(t) N_1(t) x(t) + u' N_2(t) u)\, dt \tag{1.2}$$

is minimized. Here, the prime denotes transposition and the matrices N_i are continuous, bounded and uniformly positive-definite for all $t \geqslant 0$. We note that if the controls of a motion contain linear terms, $A(t)x(t)$, they are reduced to the form of (1.1) by means of a non-singular coordinate transformation. Furthermore, under the assumptions which have been made, if the functional $J(u) < \infty$ for a certain control u, the solution of system (1.1), corresponding to this control, tends to zero as $t \to \infty$.

Let us assume that a solution of problem (1.1), (1.2) exists for the values of ε being considered and denote the optimal control by $v(t, x)$ and the corresponding Bellman function by $V(t, x)$. Sometimes, in order to emphasize the dependence of the solutions of Eqs.(1.1) on the control v, we shall denote it by $x(t, v)$.

The following lemma is used in the proof of the proposed synthesis algorithm.

Lemma 1. /5/. Let us consider the problem of the control of system (1.1) with the minimizing functional

$$\int_0^\infty K(t, x(t)), u(t, x(t))\, dt \tag{1.3}$$

where K is a continuous scalar function. Let there exist a function $V(t, x)$ which is continuously differentiable once with respect to t and x and an admissible control $v(t, x)$, $v(t, 0) = 0$ such that

$$V(t, x) \leqslant C\,|x|^2, \quad |\partial V/\partial x| \leqslant C(1 + |x|^n) \tag{1.4}$$

$$K(t, x, u) \geqslant C\,|x|^2 \tag{1.5}$$

$$\min_u \left[\frac{\partial V}{\partial t} + (\varepsilon f + Bu)' \frac{\partial V}{\partial x} + K(t, x, u)\right] = \frac{\partial V}{\partial t} + (\varepsilon f + Bv)' \frac{\partial V}{\partial x} + K(t, x, v) = 0 \tag{1.6}$$

Then, the control $v(t, x)$ solves the problem of minimizing the functional (1.3) on the trajectories of system (1.1) and, moreover, the corresponding Bellman function is equal to $V(t, x)$ and, subject to the control $v(t, x)$, the trivial solution of system (1.1) is asymptotically stable as a whole, that is, its domain of attraction is identical with the whole of the space R_n. The latter fact signifies that, for any solution $x(t, v)$ which is determined by arbitrary initial condtions, $\lim x(t, v) = 0$ as $t \to \infty$.

2. Successive approximations. We shall first present a heuristic description of the quasi-optimal stabilization algorithm. Let us write down the Bellman equation (1.6) for problem (1.1), (1.2)

$$\frac{\partial V(t, x)}{\partial t} + \varepsilon f'(t, x) \frac{\partial V(t, x)}{\partial x} + x' N_1 x = \frac{1}{4}\left(\frac{\partial V(t, x)}{\partial x}\right)' B_1 \left(\frac{\partial V(t, x)}{\partial x}\right), \quad B_1 = B N_2^{-1} B' \tag{2.1}$$

The optimal control v is equated to

$$v(t, x) = -{}^1/_2 N_2^{-1} B' \partial V(t, x)/\partial x \tag{2.2}$$

We now represent V in the form of a power series in ε

$$V(t, x) = V_0(t, x) + \varepsilon V_1(t, x) + \ldots \tag{2.3}$$

In order to determine the functions V_i, we substitute (2.3) into (2.1) and equate the coefficients accompanying like powers of ε to zero. We obtain that V_0 satisfies Eq.(2.1) when $\varepsilon = 0$ and, for V_i, $i \geqslant 1$, we have a sequence of equations which are linear in V_i

$$\frac{\partial V_i}{\partial t} + f'(t, x) \frac{\partial V_{i-1}}{\partial x} = \frac{1}{4} \sum_{j=0}^{i} \frac{\partial V_j'}{\partial x} B_1 \frac{\partial V_{i-j}}{\partial x} \tag{2.4}$$

The solution of Eq.(2.4) will be sought in the class of continuously differentiable functions which satisfy the estimates (1.4). The equation for the i-th approximation u_i to the optimal control v is

$$u_i(t,x) = -\frac{1}{2} N_2^{-1} B' \frac{\partial W}{\partial x}, \quad W = \sum_{j=0}^{i} \varepsilon^j V_j \tag{2.5}$$

Hence, a substantial part of the algorithm is associated with the need to solve problem (2.4), (2.5). We recall that, if problem (2.4), (2.5) is solvable when $\varepsilon = 0$, then $V_0 = x'P(t)x$, where the matrix $P(t)$ is the unique, bounded, positive-definite solution of the Riccati equation /1, 2/

$$P^{\cdot}(t) - P(t) B_1 P(t) + N_1(t) = 0, \quad t \geqslant 0 \tag{2.6}$$

The remaining approximations, subject to corresponding assumptions regarding the smoothness of $f(t,x)$, are given by the formula

$$V_i(t,x) = \int_t^\infty \Big[f'(s, x(s)) \frac{\partial V_{i-1}(s, x(s))}{\partial x} - \tag{2.7}$$

$$\frac{1}{4} \sum_{j=1}^{i-1} \frac{\partial V_j'(s, x(s))}{\partial x} B_1(s) \frac{\partial V_{i-j}(s, x(s))}{\partial x} \Big] ds,$$

$$i \geqslant 1; \quad V_0 = x'P(t)x$$

Here, when $i = 1$, the sum is equal to zero and $x(s)$ is the solution of Eq.(1.1) when $\varepsilon = 0$, $s \geqslant t$ with the control $u_0 = -N_2^{-1}(s) B'(s) P(s) x(s)$ and the initial condition $x(t) = x$.

3. Estimate of the zeroth approximation. The zeroth approximation control $u_0(t,x)$ is defined by expressions (2.5) and (2.6) when $i = 0$.

Let us assume that the initial control problem (1.1), (1.2) has a solution for a specified value of ε and, also, for $\varepsilon = 0$ and that the inequality

$$x'N_1 x - 2\varepsilon f'(t,x) P(t) x \geqslant C |x|^2 \tag{3.1}$$

is valid. (The different conditions of stabilizability when $\varepsilon = 0$ are given in /1, 2/).

Then, for a certain value of the constant C which is determined by the parameters of problem (1.1), (1.2)

$$0 \leqslant J(u_0) - J(v) \leqslant C\varepsilon \tag{3.2}$$

Let us prove this inequality which signifies that, when system (1.1) is controlled by means of u_0, the error in the functional (1.2) has a magnitude of the order of ε. In view of condition (3.1), the integrand of the functional

$$J_0(u) = J(u) - \varepsilon \int_0^\infty f'(t, x(t)) \frac{\partial V_0(t, x(t))}{\partial x} dt \tag{3.3}$$

is positive definite with respect to the phase coordinates, that is, a requirement of the form of (1.5)

$$x'N_1 x + u'N_2 u - \varepsilon f'(t,x) \partial V_0(t,x)/\partial x \geqslant C |x|^2 \tag{3.4}$$

is satisfied.

On the basis of Lemma 1, this means that the control $u_0(t,x)$ is a solution of the problem of minimizing the functional (3.3) on the trajectories of system (1.1) and that the corresponding Bellman function is $V_0(t,x)$, whence it also follows from (3.4) that

$$\int_0^\infty |x(t, u_0)|^2 dt \leqslant C V_0(0, x_0) \tag{3.5}$$

It follows from this inequality and expression (2.6) for u_0 that $J(u_0) < \infty$, that is, the control u_0 is also admissible for the initial problem (1.1), (1.2). Let us represent the difference from (3.2) in the form

$$J(u_0) - J(v) = \Delta_1 + \Delta_2, \ \Delta_1 = J_0(u_0) - J(v), \ \Delta_2 = J(u_0) - J_0(u_0) \tag{3.6}$$

and estimate the differences Δ_1 and Δ_2. By virtue of the optimality of the control v and the admissiblity of the control u_0

$$J(v) \leqslant J(u_0) = J_0(u_0) + \Delta_2 \tag{3.7}$$

Next, using (3.3), the boundedness of P and the assumptions regarding f, we have

$$\Delta_2 \leqslant 2\varepsilon \int_0^\infty |f'(t, x(t, u_0)) P(t) x(t, u_0)| dt \leqslant 2\varepsilon C \int_0^\infty |x(t, u_0)|^2 dt \leqslant \varepsilon C_1 \tag{3.8}$$

Hence, by virtue of (3.7)

$$J(v) \leqslant J_0(u_0) + \varepsilon C_1 \tag{3.9}$$

Similarly, in view of the optimality of the control u_0 in problem (1.1), (3.3) and by virtue of the uniform positive-definiteness of the matrix N_1 and the estimate (3.9), we have

$$J_0(u_0) \leqslant J(v) + |J_0(v) - J(v)| \leqslant J(v) + \varepsilon C_2 \tag{3.10}$$

Comparing relationships (3.9) and (3.10), we conclude that $|\Delta_1| \leqslant \varepsilon C$. From this inequality and (3.8), we have (3.2) by virtue of (3.6).

4. Estimates of the higher approximations. In considering the i-th approximation, u_i, to the optimal control, it is postulated that there exist continuously differentiable functions $V_j, j \leqslant i$ which satisfy Eq.(2.4) and, also, the estimates

$$|W(t,x)| \leqslant C|x|^2, \quad |\partial W(t,x)/\partial x| \leqslant C|x|, \quad j \leqslant i \tag{4.1}$$

It follows from definition (2.5) of the controls $u_j(t,x)$ and (4.1) that $u_j(t, 0) \equiv 0$.

The method employed above in proving the estimate for the zeroth approximation (3.2) consisted of the fact that the zeroth approximation control, u_0, is to be interpreted as the optimal initial system (1.1) with a minimized functional which differs from the initial functional (1.2) a quantity of the order of magnitude of ε. We also use this approach when investigating the controls $u_i(t,x)$, $i \geqslant 1$ which are defined by equality (2.6). In order to construct an auxiliary stabilization problem in which $u_i(t,x)$ will be the optimal control, we write down the equation for the function $W(t,x)$ from (2.6). After multiplying Eq.(2.4) by ε^i and summing the resulting equation over i, we have

$$\begin{gathered}\frac{\partial W}{\partial t} + \varepsilon f'(t,x)\frac{\partial W}{\partial x} + x'N_1x + \delta_i\varepsilon^{i+1} = \\ \frac{1}{4}\left(\frac{\partial W}{\partial x}\right)' B_1\left(\frac{\partial W}{\partial x}\right) \\ \delta_i = -f'\frac{\partial V_i}{\partial x} + \frac{1}{4}\sum_{l=1}^{i}\sum_{j=0}^{l-1}\varepsilon^j\left(\frac{\partial V_l}{\partial x}\right)' B_1\left(\frac{\partial V_{i+1+j-l}}{\partial x}\right)\end{gathered} \tag{4.2}$$

By analogy with (3.1), we shall assume in the case being considered that problem (1.1), (1.2) has a solution for the specified ε and, also, for $\varepsilon = 0$ and that the inequality

$$x'N_1x - \varepsilon^{i+1}\delta_i \geqslant C|x|^2 \tag{4.3}$$

holds.

Under assumption (4.3), the functional

$$J_i(u) = J(u) + \varepsilon^{i+1}\int_0^T \delta_i(t, x(t,u))\,dt \tag{4.4}$$

is positive-definite with respect to the phase coordinates. Hence, by virtue of Lemma 1, and taking account of Eq.(4.2) and the estimates (4.1), a control u_i will be optimal in problem (1.1), (4.4) with a Bellman function equal to W. It follows from this and (4.3) that

$$\int_0^\infty |x(t,u_i)|^2\,dt \leqslant CW(0,x_0) \tag{4.5}$$

Additionally, $2u_i(t,x) = -N_2^{-1}(t)B'(t)\partial W(t,x)/\partial x$. Hence, on the basis of (4.1) and (2.6), we have $|u_i(t,x)| \leqslant C|x|$. This means that, when account is taken of (4.5), $J(u_i) < \infty$, that is, u_i is an admissible control in the stabilization problem (1.1), (1.2). The subsequent proof of the estimates of the i-th approximations is analogous to the proof of inequality (3.2). It can be shown that

$$0 \leqslant J(u_i) - J(v) \leqslant C\varepsilon^{i+1} \tag{4.6}$$

It is clear that $J(u_i) - J(v) = [J_i(u_i) - J(v)] + [J(u_i) - J_i(u_i)]$. Let us estimate the differences on the right hand side of the latter relationship. By virtue of (4.4) and (4.5), we have

$$J(v) \leqslant J(u_i) \leqslant J_i(u_i) + |J(u_i) - J_i(u_i)| \leqslant J_i(u_i) + \varepsilon^{i+1}C\gamma_i, \quad \gamma_i = \int_0^\infty |x(t,u_i)|^2\,dt \tag{4.7}$$

In order to prove the inequality which is the opposite of (4.7), we use the optimality of the control u_i in stabilization problem (1.1) and (4.3). Then,

$$J_i(u_i) \leqslant J_i(v) \leqslant J(v) + |J_i(v) - J(v)| \leqslant J(v) + \varepsilon^{i+1}C\int_0^\infty |x(t,v)|^2\,dt \leqslant J(v) + \varepsilon^{i+1}C_1W(0,x_0) \tag{4.8}$$

It follows from the inequalities (4.7) and (4.8) that

$$|J_i(u_i) - J(v)| \leqslant \varepsilon^{i+1}C\gamma_i \tag{4.9}$$

Furthermore, it has been established when deriving the estimate (4.7) that

$$|J(u_i) - J_i(u_i)| \leqslant \varepsilon^{i+1}\,C\,C_i \tag{4.10}$$

Relationships (4.9) and (4.10) signify that

$$0 \leqslant J(u_i) - J(v) \leqslant \varepsilon^{i+1}\,C\,C_i$$

An estimate of the i-th approximation of (4.6) is thereby established.

We note that problems involving the control of systems with a small parameter in a finite interval of time are considered in /6/.

Remark. We present the sufficient conditions for the estimates of (4.1) to be valid. Let us assume that problem (1.1), (1.2) is solvable when $\varepsilon = 0$, that the function $f(t,x)$ and its derivatives $\partial^j f(t,x)/\partial x^j$ are continuous and that $|\partial^j f/\partial x^j| \leqslant C$, $j = 1,\ldots,i$; $i \geqslant 1$. Then, a unique solution of Eq.(2.4) exists which satisfies the estimates (4.1) and (4.4). This solution is representable in the form of (2.7).

5. Adaptive stabilization. Let us now assume that the actual value of the parameter ε in (1.1) is unknown. The stabilization problem (which is now referred to as an adaptive stabilization problem) involves the choice of a control law such that, under this law, the trivial solution of system (1.1) is asymptotically stable on the whole. We shall seek adaptive control, which solves the problem of adaptive stabilization, in the form $B\omega(t,x) - k(t)f(t,x)$. The function $\omega(t,x)$ and the scalar coefficient $k(t)$ of the auxiliary control contour are selected in such a way as to ensure the stability of the trivial solution of the system

$$x^{\cdot}(t) = (\varepsilon - k(t))\,f(t,x(t)) + B(t)\,\omega(t,x(t)) \tag{5.1}$$

Lemma 2. For a certain fixed value of ε_0, let a control $\omega_0(t,x)$, $\omega_0(t,0) = 0$ be found for which the trivial solution of the equation

$$y^{\cdot}(t) = \varepsilon_0 f(t,y(t)) + B\omega_0(t,y(t)),\ t \geqslant 0$$

is asymptotically stable uniformly with respect to the initial data. Then, the problem of adaptive stabilization is solvable for any ε.

Proof. The existence of a Lyapunov function $V(t,x)$ follows from the conditions of Lemma 2 and the inversion theorem in the theory of stability /7/. The function V is strictly positive, has an infinitely small higher limit and the sum $R(t,x) = V_t(t,x) + V_x'(t,x)\ (\varepsilon_0 f(t,x) + B\omega_0(t,x))$ is strictly negative. Let us define the control coefficient $k(t)$ in (5.1) by the relationship

$$\begin{aligned} &k^{\cdot}(t) = -{}^1\!/_2 f'(t,x(t))\,V_x(t,x(t)),\ t \geqslant 0 \\ &V_x = \partial V/\partial x,\ k(0) = 0 \end{aligned} \tag{5.2}$$

and consider the function $R_1(t,x) = V(t,x) + (\varepsilon - \varepsilon_0 - k(t))^2$. For the complete derivative of this function along the trajectories of (5.1), (5.2), we have $R_1^{\cdot} = R$. This means, by virtue of the theorem concerning stability along parts of variables /8/, that the trivial solution of system (5.1) is asymptotically stable as a whole. The control ω_0 and the regulator (5.2) thereby ensure the stability of system (5.1) for any a priori unknown value of the parameter ε.

Lemma 2 reduces the question concerning the adaptive stabilization of system (1.1) to the choice of the Lyapunov function V and the control ω_0. When this is so, use can be made of the construction in Sects.1-3. In particular, if system (1.1) can be stabilized when $\varepsilon = 0$, it is possible to put $V = V_0$ and $\omega_0 = u_0$ where $V_0 = x'P(t)x$ and $u_0 = -N_2^{-1}B'Px$. If the conditions in Sect.4 are satisfied when $\varepsilon = \varepsilon_0$, one can put $V = W$, $\omega_0 = u_i$, where the functions W and u_i are given by Eqs.(2.5) when $\varepsilon = \varepsilon_0$. In both cases, the coefficient $k(t)$ is defined by Eq.(5.2). Meanwhile, when actual systems are considered, it is possible with the help of Lemma 2, using their specific characteristics, to construct a stabilizing control which does not depend on all of the phase coordinates. The latter fact is important from the point of view of the minimization of the number of position sensing devices. The solution of the problem on the adaptive stablization of a specified programmed motion of system (1.1) can be formulated in terms which are analogous to Lemma 2. We note that the problem of the optimal stabilization when there are constantly acting perturbations is discussed in /10/.

Examples. 1°. Let us consider a single-link manipulator consisting of an absolutely solid, homogeneous, linear rod of length L and mass M. One end of this rod is connected to an ideal cylindrical articulation O with a fixed base while the load of mass m which is to be moved is rigidly clamped to the other end of the rod. A control moment u is applied to the axis of the articulation O. The motion takes place in a vertical plane in a gravitational force field. The axis of the articulation O is perpendicular to the plane of motion. The equations of motion have the form

$$L^2 m_1 \varphi^{\cdot\cdot} + \alpha\varphi^{\cdot} + gL\,(m + M/2)\sin\varphi = u$$
$$\varphi(0) = \varphi_0,\ \varphi^{\cdot}(0) = \varphi_0^{\cdot};\ m_1 = m + M/3$$

Here, φ is the angle between the axis of the rod and a vertical line lying in the plane of motion and perpendicular to the axis of the articulation, g is the gravitational constant, and α is the coefficient of viscous friction. The value of α is generally only known imprecisely (/9/, p.215). Below, it is only assumed, regarding α, that $\alpha \geqslant \alpha_0 > 0$ and that the actual value of α is unknown. This problem involves the selection of that control moment u under the action of which the load m is moved from an arbitrary initial position $\varphi_0, \varphi_0^{\cdot}$ to the origin of coordinates $(\varphi = 0, \varphi^{\cdot} = 0)$. Since the coefficient of friction α, is unknown, the time taken to reach the origin of coordinate is also unknown. Hence, the synthesis of that control u under the action of which

$$\lim \varphi^{\cdot}(t) = 0,\ \lim \varphi(t) = 0,\ t \to \infty \tag{5.3}$$

is a possible formalization of the problem which has been posed.

Let us specify the control u in the form $u = -b_1\varphi(t), b_1 > 0$. We note that such control can be realized using standard proportional controls. Let us write the equations of motion under this control in the form

$$\varphi_1^{\cdot} = \varphi_2,\ \varphi_2^{\cdot} = -a_1\varphi_2 - a_2\sin\varphi_1 - b\varphi_1 \tag{5.4}$$
$$a_1 = \alpha L^{-2} m_1^{-1},\ a_2 = gL^{-1}(m + M/2)\, m_1^{-1},\ b = b_1 L^{-2} m_1^{-1}$$

We shall take the following function

$$V = 2b\,\varphi_1^2 + \varphi_2^2 + (\varphi_2 + a_1\varphi_1)^2 \tag{5.5}$$

as the Lyapunov function from Lemma 2 for system (5.4).

For any $\delta > 0$, the derivative of the function V along the trajectories of system (5.4) satisfies the relationship

$$V^{\cdot} \leqslant -2a_1\varphi_1^2(b - a_2 - a_2\delta^{-1}) - 2\varphi_2^2(a_1 - \delta a_2)$$

Let us choose and fix δ such that $\alpha_0 L^{-2} m_1^{-1} - \delta a_2 > 0$ and, after this, select b from the condition $b - a_2 - a_2\delta^{-1} > 0$. Then $V^{\cdot}$ will be negative-definite, whence (5.4) also follows from (5.5).

2°. Let us consider the problem of the stabilization of an object which corresponds, for example, to robots of the "Cyclon" type. The hand of the robot of length L and mass M is set in motion by means of double acting pneumatic cylinders via a transmission mechanism with a shoulder l. A load of unknown mass m is situated in the claw of the hand. Let us put

$$x_1^{\cdot} = \varphi^{\cdot},\ x_2 = \varphi,\ x_3 = P,\ a_1 = 2Fl\,(m_1L^2)^{-1}$$
$$a_3 = 4P_1FlV_1^{-1},\ b = 2RTV_1^{-1},\ m_1 = m + M/3$$

Here, φ is the angular motion of the hand of the manipulator, P is the current pressure in the pneumatic cylinders, F is the area of the piston, R is the universal gas constant, T is the absolute temperature of the gas, V_1 is the volume of the pneumatic cylinder, P_1 is the average pressure in it, g is the mass flow rate into the cavities of the pneumatic cylinders, and $f(\varphi^{\cdot}) = -\alpha\varphi^{\cdot}$ is the force due to viscous friction in the device used with an unknown coefficient of friction $\alpha \geqslant 0$.

The equations of motion /11/ then take the form

$$x_1^{\cdot} = a_1x_3 - \alpha x_1,\ x_2^{\cdot} = x_1,\ x_3^{\cdot} = -a_3x_1 + bu \tag{5.6}$$

The problem involves the synthesis of a control u under the action of which

$$\lim(|x_1(t)| + |x_2(t)| + |x_3(t)|) = 0,\ t \to \infty \tag{5.7}$$

for any initial values of the variables x_i, which corresponds to the problem of the transfer of an unknown load m from an arbitrary position to the origin of coordinates. Let us put

$$bu = -b_1x_2 - b_2x_3 \tag{5.8}$$

and choose the constants $b_i > 0$ such that condition (5.7) is satisfied. We now introduce the Lyapunov function

$$V = (x_3 + b_2a_1^{-1}x_1)^2 + (x_3 + b_2a_1^{-1}x_1 + a_3x_2)^2 + x_3^2 + 2a_1^{-1}x_1^2(a_3 + \alpha\, b_2a_1^{-1}) + b_2a_1^{-1}(2b_1 + \alpha a_3)\,x_2^2 \tag{5.9}$$

into the treatment. The total derivative of the function V along the trajectories of system (5.6) under the action of control (5.8) is

$$V^{\cdot} = -x_1^2(2a_3b_2a_1^{-1} + 4b_2^2a_1^{-1}\alpha + 4a_3a_1^{-1}\alpha) - 2a_3b_1x_2^2 - 2b_2x_3^2 - 6b_1x_2x_3 \tag{5.10}$$

But, for any $\delta > 0$,

$$6b_1x_2x_3 \leqslant 3b_1\ (x_2^2\delta + x_3^2\delta^{-1})$$

Let us choose and fix $\delta > 0$ such that

$$2a_3 - 3\delta > 0 \tag{5.11}$$

After this, we take any $b_1 > 0$ and b_2 which satisfies the condition

$$2b_2 > 3b_1\delta^{-1} \tag{5.12}$$

It follows from (5.10)-(5.12) that $V^{\cdot}$ is a strictly negative function. Condition (5.7) follows from this fact and from (5.9). So, for the selected b_1 and b_2, the control (5.8) solves the problem which has been posed.

We note that, if it is required that system (5.6) be transferred from an arbitrary initial position to a final position $(0, \bar{x}_2, 0)$ with an arbitrary $\bar{x}_2$ and a zero velocity x_1 at the end, one may use the control $bu = -b_1x_2 - b_2x_3 + b_1\bar{x}_2$ with the previous values of b_1 and b_2.

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